

Exhibit

A

Thomas H. Cormen
Charles E. Leiserson
Ronald L. Rivest

Introduction to Algorithms

BEST AVAILABLE COPY

The MIT Press
Cambridge, Massachusetts London, England

McGraw-Hill Book Company
New York St. Louis San Francisco Montreal Toronto

Eighth printing, 1992

This book is one of a series of texts written by faculty of the Electrical Engineering and Computer Science Department at the Massachusetts Institute of Technology. It was edited and produced by The MIT Press under a joint production-distribution agreement with the McGraw-Hill Book Company.

Ordering Information:

North America

Text orders should be addressed to the McGraw-Hill Book Company. All other orders should be addressed to The MIT Press.

Outside North America

All orders should be addressed to The MIT Press or its local distributor.

©1990 by The Massachusetts Institute of Technology

All rights reserved. No part of this book may be reproduced in any form or by any electronic or mechanical means (including photocopying, recording, or information storage and retrieval) without permission in writing from the publisher.

This book was printed and bound in the United States of America.

Library of Congress Cataloging-in-Publication Data

Cormen, Thomas H.

Introduction to Algorithms / Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest.

p. cm.—(The MIT electrical engineering and computer science series)

Includes bibliographical references.

ISBN 0-262-03141-8 (MIT Press).—ISBN 0-07-013143-0 (McGraw-Hill)

1. Electronic digital computers—Programming. 2. Algorithms.

I. Leiserson, Charles Eric. II. Rivest, Ronald L. III. Title

IV. Series.

QA76.6.C662 1989

005.1—dc20

89-13027

CIP

11

Elementary Data Structures

In this chapter, we examine the representation of dynamic sets by simple data structures that use pointers. Although many complex data structures can be fashioned using pointers, we present only the rudimentary ones: stacks, queues, linked lists, and rooted trees. We also discuss a method by which objects and pointers can be synthesized from arrays.

11.1 Stacks and queues

Stacks and queues are dynamic sets in which the element removed from the set by the **DELETE** operation is prespecified. In a *stack*, the element deleted from the set is the one most recently inserted: the stack implements a *last-in, first-out*, or *LIFO*, policy. Similarly, in a *queue*, the element deleted is always the one that has been in the set for the longest time: the queue implements a *first-in, first-out*, or *FIFO*, policy. There are several efficient ways to implement stacks and queues on a computer. In this section we show how to use a simple array to implement each.

Stacks

The **INSERT** operation on a stack is often called **PUSH**, and the **DELETE** operation, which does not take an element argument, is often called **POP**. These names are allusions to physical stacks, such as the spring-loaded stacks of plates used in cafeterias. The order in which plates are popped from the stack is the reverse of the order in which they were pushed onto the stack, since only the top plate is accessible.

As shown in Figure 11.1, we can implement a stack of at most n elements with an array $S[1..n]$. The array has an attribute $\text{top}[S]$ that indexes the most recently inserted element. The stack consists of elements $S[1..\text{top}[S]]$, where $S[1]$ is the element at the bottom of the stack and $S[\text{top}[S]]$ is the element at the top.

When $\text{top}[S] = 0$, the stack contains no elements and is *empty*. The stack can be tested for emptiness by the query operation **STACK-EMPTY**. If an

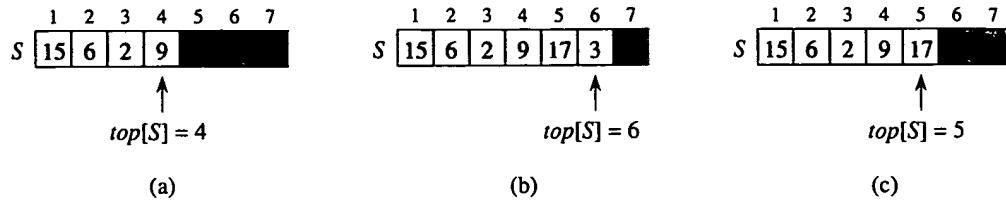


Figure 11.1 An array implementation of a stack S . Stack elements appear only in the lightly shaded positions. (a) Stack S has 4 elements. The top element is 9. (b) Stack S after the calls $\text{PUSH}(S, 17)$ and $\text{PUSH}(S, 3)$. (c) Stack S after the call $\text{POP}(S)$ has returned the element 3, which is the one most recently pushed. Although element 3 still appears in the array, it is no longer in the stack; the top is element 17.

empty stack is popped, we say the stack *underflows*, which is normally an error. If $\text{top}[S]$ exceeds n , the stack *overflows*. (In our pseudocode implementation, we don't worry about stack overflow.)

The stack operations can each be implemented with a few lines of code.

STACK-EMPTY(S)

```

1 if top[S] = 0
2   then return TRUE
3   else return FALSE

```

PUSH(S, x)

```

1  $top[S] \leftarrow top[S] + 1$ 
2  $S[top[S]] \leftarrow x$ 

```

$\text{Pop}(S)$

```

1 if STACK-EMPTY( $S$ )
2   then error “underflow”
3   else  $top[S] \leftarrow top[S] - 1$ 
4   return  $S[top[S] + 1]$ 

```

Figure 11.1 shows the effects of the modifying operations `PUSH` and `POP`. Each of the three stack operations takes $O(1)$ time.

Queues

We call the **INSERT** operation on a queue **ENQUEUE**, and we call the **DELETE** operation **DEQUEUE**; like the stack operation **POP**, **DEQUEUE** takes no element argument. The **FIFO** property of a queue causes it to operate like a line of people in the registrar's office. The queue has a **head** and a **tail**. When an element is enqueued, it takes its place at the tail of the queue, like a newly arriving student takes a place at the end of the line. The ele-

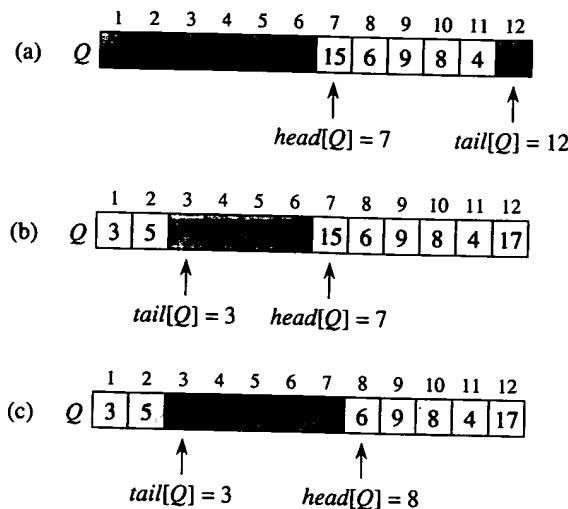


Figure 11.2 A queue implemented using an array $Q[1..12]$. Queue elements appear only in the lightly shaded positions. (a) The queue has 5 elements, in locations $Q[7..11]$. (b) The configuration of the queue after the calls $\text{ENQUEUE}(Q, 17)$, $\text{ENQUEUE}(Q, 3)$, and $\text{ENQUEUE}(Q, 5)$. (c) The configuration of the queue after the call $\text{DEQUEUE}(Q)$ returns the key value 15 formerly at the head of the queue. The new head has key 6.

ment dequeued is always the one at the head of the queue, like the student at the head of the line who has waited the longest. (Fortunately, we don't have to worry about computational elements cutting into line.)

Figure 11.2 shows one way to implement a queue of at most $n - 1$ elements using an array $Q[1..n]$. The queue has an attribute $head[Q]$ that indexes, or points to, its head. The attribute $tail[Q]$ indexes the next location at which a newly arriving element will be inserted into the queue. The elements in the queue are in locations $head[Q], head[Q]+1, \dots, tail[Q]-1$, where we "wrap around" in the sense that location 1 immediately follows location n in a circular order. When $head[Q] = tail[Q]$, the queue is empty. Initially, we have $head[Q] = tail[Q] = 1$. When the queue is empty, an attempt to dequeue an element causes the queue to underflow. When $head[Q] = tail[Q] + 1$, the queue is full, and an attempt to enqueue an element causes the queue to overflow.

In our procedures ENQUEUE and DEQUEUE , the error checking for underflow and overflow has been omitted. (Exercise 11.1-4 asks you to supply code that checks for these two error conditions.)

$\text{ENQUEUE}(Q, x)$

- 1 $Q[tail[Q]] \leftarrow x$
- 2 if $tail[Q] = length[Q]$
- 3 then $tail[Q] \leftarrow 1$
- 4 else $tail[Q] \leftarrow tail[Q] + 1$

**This Page is Inserted by IFW Indexing and Scanning
Operations and is not part of the Official Record**

BEST AVAILABLE IMAGES

Defective images within this document are accurate representations of the original documents submitted by the applicant.

Defects in the images include but are not limited to the items checked:

BLACK BORDERS

IMAGE CUT OFF AT TOP, BOTTOM OR SIDES

FADED TEXT OR DRAWING

BLURRED OR ILLEGIBLE TEXT OR DRAWING

SKEWED/SLANTED IMAGES

COLOR OR BLACK AND WHITE PHOTOGRAPHS

GRAY SCALE DOCUMENTS

LINES OR MARKS ON ORIGINAL DOCUMENT

REFERENCE(S) OR EXHIBIT(S) SUBMITTED ARE POOR QUALITY

OTHER: _____

IMAGES ARE BEST AVAILABLE COPY.

As rescanning these documents will not correct the image problems checked, please do not report these problems to the IFW Image Problem Mailbox.